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Suppression of Proton Decay in the Missing-Partner Model for Supersymmetric SU(5) GUT¹

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Abstract

The Peccei-Quinn symmetric extension of the missing-partner model in the supersymmetric SU(5) grand unified model is consistent with the observed stability of the proton, even in the large $\tan\beta$ region ($\simeq 50 - 60$) expected from the Yukawa unification. Moreover, the SU(5) gauge coupling constant remains small enough for the perturbative description of GUT's below the gravitational scale.

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1 Introduction

Supersymmetry (SUSY) is introduced to protect the weak scale from the radiative corrections [1]. Since supersymmetry is a symmetry between bosons and fermions, the minimal supersymmetric extension of standard model (MSSM) introduces scalar bosons with baryon or lepton numbers, which are called as squarks or sleptons. Therefore, there may be baryon or lepton number violating operators with the dimensions four or five, which do not exist in the standard model. The dimension-four operators induce various phenomenological difficulties, especially, instability of proton. Therefore, they are expected to be forbidden by a symmetry, and a Z_2 symmetry called as R parity is often assumed phenomenologically.

Quark-quark-squark-slepton is a dimension-five operator violating both baryon and lepton numbers. This is suppressed by a mass parameter, which we call as Λ from now on. The magnitude of Λ depends on the origin of the dimension-five operator. In the supergravity model Λ is expected to be $m_{pl}/\sqrt{8\pi}(= 2.4 \times 10^{18}\text{GeV})$, and then the proton lifetime is $10^{(26-28)}$ years. This value is lower than the experimental lowerbound by the magnitude of $10^{-(4-6)}$. The dimension-five operator is also expected to be forbidden by a symmetry.

One of the candidates for such a symmetry is the Peccei-Quinn (PQ) symmetry [2], which is a solution of strong CP problem. This is because a U(1) symmetry forbidding the dimension-five operator must have a triangle anomaly. On the other hand, the breaking scale of the PQ symmetry (M_{PQ}) is constrained from astronomy and cosmology as following [3],

$$10^{10}\text{GeV} \leq M_{PQ} \leq 10^{13}\text{GeV}. \quad (1)$$

Therefore, the PQ symmetry can not forbid the dimension-five operator completely, however, can suppress it by a factor of (M_{PQ}/Λ) . If Λ is 2.4×10^{18} GeV, the proton lifetime can reach $10^{(36-46)}$ years, and we can avoid the constraint from the observation. The PQ symmetry is expected not only from the strong CP problem, but also from the stability of proton.

Next, we will extend the SU(5) supersymmetric grand unified theory (SU(5) SUSY GUT)[4], which is very interesting from both experimental and theoretical

points of view, to have the PQ symmetry. In the minimal SU(5) SUSY GUT the dimension-five operator comes from Yukawa couplings giving masses to quarks and leptons. Quarks and leptons are embedded in $\psi_i[\mathbf{10}]$ and $\phi_i[\mathbf{5}^*]$ ($i=1,2,3$). A pair of SU(2)_L-doublet Higgses in MSSM, H_f and \overline{H}_f , is in $H[\mathbf{5}]$ and $\overline{H}[\mathbf{5}^*]$ as

$$\begin{aligned} H^A &= \left(H_c, H_c, H_c, H_f, H_f \right)^t, \\ \overline{H}_A &= \left(\overline{H}_c, \overline{H}_c, \overline{H}_c, \overline{H}_f, \overline{H}_f \right)^t, \end{aligned} \quad (2)$$

where $A(=1 \cdots 5)$ is a SU(5) fundamental representation index. Here, two color-triplet Higgses, H_c and \overline{H}_c , have to be introduced in H and \overline{H} . The SU(5) symmetric Yukawa couplings are given as

$$W_{\text{Yukawa}} = \frac{1}{4} f_u^{ij} \epsilon_{ABCDE} \psi_i^{(AB)} \psi_j^{(CD)} H^E + \sqrt{2} f_d^{ij} \psi_i^{(AB)} \phi_{jA} \overline{H}_B, \quad (3)$$

where ϵ_{ABCDE} is a fifth antisymmetric tensor. The dimension-five operator is generated by an exchange of the color-triplet Higgses through these Yukawa couplings [5]. The present lower bound of the color-triplet Higgs mass from the negative search of proton decay has already reached at 2×10^{16} GeV[6, 7, 8], and the minimal SU(5) SUSY GUT is also strongly constrained now.

If the minimal SU(5) SUSY GUT is extended to have the PQ symmetry, there may be no problem of the dimension-five operator. However, the color-triplet Higgs mass must vanish, and proton can decay very rapidly again through the dimension-six operator induced by an exchange of color-triplet Higgs boson. In order to preserve the PQ symmetry in the Yukawa couplings (3), each chiral multiplets are transformed under the PQ symmetry as

$$\begin{aligned} \psi_i &\rightarrow e^{i\alpha} \psi_i, \\ \phi_i &\rightarrow e^{i\beta} \phi_i, \\ H &\rightarrow e^{-2i\alpha} H, \\ \overline{H} &\rightarrow e^{-i(\alpha+\beta)} \overline{H}, \end{aligned} \quad (4)$$

where $3\alpha + \beta \neq 0$ to forbid the dimension-five operator. Therefore, H and \overline{H} can not have an SU(5) symmetric mass term, and the color-triplet Higgs masses must be zero.

A simple way for the color-triplet Higgses to have a GUT-scale mass is to introduce new **5**- and **5***-dimension Higgses (H' and \overline{H}') with the $U(1)_{PQ}$ charges opposite to \overline{H} and H [9]. However, this extension violates a experimental success of gauge coupling unification. In this extension there is another pair of $SU(2)_L$ -doublet Higgses in H' and \overline{H}' , and these can acquire a mass only from the vacuum expectation value breaking the PQ symmetry. These extra $SU(2)_L$ -doublet Higgses give extra corrections to the $SU(2)_L$ and $U(1)_Y$ gauge coupling constants. Therefore, this extension breaks the successful gauge coupling unification. To reproduce the gauge coupling unification, we have to introduce many particles with the masses at the GUT scale so that the threshold corrections to the three gauge coupling constants can compensate those from these extra $SU(2)_L$ -doublet Higgses.

In next section, we will propose the missing-partner model with the PQ symmetry. Since this model gives the large threshold corrections to the three gauge coupling constants at the GUT scale generically [10], the success of gauge coupling unification can be acquired naturally, and the PQ symmetry can suppress sufficiently the proton decay through the dimension-five operator[11].

2 The Missing-Partner Model with the Peccei-Quinn symmetry

The missing-partner model was proposed for the $SU(2)_L$ -doublet Higgses in MSSM not to receive, group-theoretically, a mass from the $SU(5)$ -breaking vacuum expectation value[12]. This model has three Higgses, Σ [**75**], θ [**50**], and $\overline{\theta}$ [**50***] with H and \overline{H} that we have mentioned above. The superpotential has following terms,

$$\begin{aligned}
W = & G_H H^A \Sigma_{(FG)}^{(BC)} \theta^{(DE)(FG)} \epsilon_{ABCDE} + G_{\overline{H}} \overline{H}_A \Sigma_{(BC)}^{(FG)} \overline{\theta}_{(DE)(FG)} \epsilon^{ABCDE} \\
& + M_{75} \Sigma_{(CD)}^{(AB)} \Sigma_{(AB)}^{(CD)} - \frac{1}{3} \lambda_{75} \Sigma_{(EF)}^{(AB)} \Sigma_{(AB)}^{(CD)} \Sigma_{(CD)}^{(EF)} \\
& + W_{\text{Yukawa}}
\end{aligned} \tag{5}$$

with a mass term of θ and $\overline{\theta}$ ($M_{50} \overline{\theta}_{(AB)(CD)} \theta^{(AB)(CD)}$). The **75**-dimension Higgs Σ acquires a vacuum expectation value and it breaks the $SU(5)$ symmetry. There can be no $H \Sigma \overline{H}$ term due to the $SU(5)$ gauge symmetry, and the **50**-dimension Higgses,

θ and $\bar{\theta}$, have $SU(3)_C$ triplet components, however, no $SU(2)_L$ -doublet component. Therefore, $\langle \Sigma \rangle$ can give masses to the color-triplet Higgses, however, not to the $SU(2)_L$ -doublet Higgses.

The missing-partner model has two different behaviors of the running gauge coupling constants from the minimal $SU(5)$ SUSY GUT since this model has large dimension Higgses. First, the $SU(5)$ gauge coupling constant blows up rapidly above the mass of θ and $\bar{\theta}$, M_{50} . If a perturbative picture is expected not to be broken below the gravitational scale, M_{50} should be at least above the gravitational scale. However, in that case the color-triplet Higgses inducing proton decay have a smaller mass than the GUT scale since they have a see-saw type mass matrix. In a case where M_{50} is $2.4 \times 10^{18} \text{ GeV}$, the color-triplet Higgs mass becomes $10^{(14-15)} \text{ GeV}$, that is completely excluded experimentally.

Second, the **75**-dimension Higgs Σ gives large threshold corrections to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge coupling constants. The components of Σ acquire different masses from each others due to its own vacuum expectation value as following table [10].

$(SU(3)_C \times SU(2)_L \times U(1)_Y)$	mass
$(\mathbf{8}, \mathbf{3}, 0)$	M_Σ
$(\mathbf{3}, \mathbf{1}, \frac{5}{3}), (\bar{\mathbf{3}}, \mathbf{1}, -\frac{5}{3})$	$\frac{4}{5}M_\Sigma$
$(\mathbf{6}, \mathbf{2}, \frac{5}{6}), (\bar{\mathbf{6}}, \mathbf{2}, -\frac{5}{6})$	$\frac{2}{5}M_\Sigma$
$(\mathbf{1}, \mathbf{1}, 0)$	M_Σ
$(\mathbf{8}, \mathbf{1}, 0)$	$\frac{1}{5}M_\Sigma$
$(\mathbf{3}, \mathbf{2}, -\frac{5}{6}), (\bar{\mathbf{3}}, \mathbf{2}, \frac{5}{6})$	Nambu-Goldstone multiplets

Here $M_\Sigma = 5M_{75}$. This mass splitting contributes to the differences of the three gauge coupling constants. This was needed surely when we would extend the minimal $SU(5)$ SUSY GUT to have the PQ symmetry.

The missing-partner model has a more severe problem for the stability of proton if we assume the perturbative picture below the gravitational scale. However, since there are large threshold corrections to the gauge coupling constants at the GUT

scale, the problem is expected to be solved completely by extending this model to have the PQ symmetry.

To preserve the PQ symmetry in the superpotential (5), Σ , θ , and $\bar{\theta}$ are transformed as

$$\begin{aligned}\theta(\mathbf{50}) &\rightarrow e^{2i\alpha}\theta(\mathbf{50}), \\ \bar{\theta}(\bar{\mathbf{50}}) &\rightarrow e^{i(\alpha+\beta)}\bar{\theta}(\bar{\mathbf{50}}), \\ \Sigma(\mathbf{75}) &\rightarrow \Sigma(\mathbf{75}),\end{aligned}\tag{6}$$

with the other chiral multiplets transformed as Eqs. (4). Here, we have to introduce newly $H'[\mathbf{5}]$, $\bar{H}'[\mathbf{5}^*]$, $\theta'[\mathbf{50}]$, and $\bar{\theta}'[\mathbf{50}^*]$ with the $U(1)_{PQ}$ charges opposite to the corresponding chiral multiplets. This is also because these Higgses have the PQ symmetric masses. We add a new superpotential to Eq. (5),

$$\begin{aligned}W' &= G'_H H'^A \Sigma_{(FG)}^{(BC)} \theta'^{(DE)(FG)} \epsilon_{ABCDE} + G'_{\bar{H}} \bar{H}'_A \Sigma_{(BC)}^{(FG)} \bar{\theta}'_{(DE)(FG)} \epsilon^{ABCDE} \\ &\quad + M_1 \bar{\theta}'_{(AB)(CD)} \theta^{(AB)(CD)} + M_2 \bar{\theta}_{(AB)(CD)} \theta'^{(AB)(CD)}.\end{aligned}\tag{7}$$

To avoid that the $SU(5)$ gauge coupling constant blows up below the gravitational scale $M_{pl}/\sqrt{8\pi}$, we assume

$$M_1, M_2 \gtrsim 10^{18} \text{ GeV}.\tag{8}$$

In the following, we take $M_1 = M_2 = M_{pl}/\sqrt{8\pi} (\equiv 2.4 \times 10^{18} \text{ GeV})$ for simplicity. Then, we have four Higgses, H , \bar{H} , H' , \bar{H}' , and one Higgs Σ much below the gravitational scale.

The $\mathbf{75}$ -dimension Higgs Σ has the following vacuum expectation value that causes the breaking $SU(5) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$,

$$\begin{aligned}\langle \Sigma \rangle_{(\gamma\delta)}^{(\alpha\beta)} &= \frac{1}{2} \left\{ \delta_\gamma^\alpha \delta_\delta^\beta - \delta_\delta^\alpha \delta_\gamma^\beta \right\} V_\Sigma, \\ \langle \Sigma \rangle_{(cd)}^{(ab)} &= \frac{3}{2} \left\{ \delta_c^a \delta_d^b - \delta_d^a \delta_c^b \right\} V_\Sigma, \\ \langle \Sigma \rangle_{(b\beta)}^{(a\alpha)} &= -\frac{1}{2} \left\{ \delta_b^a \delta_\beta^\alpha \right\} V_\Sigma,\end{aligned}\tag{9}$$

where

$$V_\Sigma = \frac{3}{2} \frac{M_{75}}{\lambda_{75}}.\tag{10}$$

Here, $\alpha, \beta \dots$ are the $SU(3)_C$ indices and $a, b \dots$ the $SU(2)_L$ indices. This vacuum expectation value generates masses for the color-triplet Higgses as (after integrating out the heavy fields, $\theta, \bar{\theta}'$ and $\theta', \bar{\theta}$),

$$M_{H_c} H_c^\alpha \bar{H}'_{c\alpha} + M_{\bar{H}_c} H'^\alpha_c \bar{H}_{c\alpha}, \quad (11)$$

with

$$M_{H_c} \simeq 48 G_H G'_H \frac{V_\Sigma^2}{M_1}, \quad M_{\bar{H}_c} \simeq 48 G_H G'_H \frac{V_\Sigma^2}{M_2}. \quad (12)$$

For $V_\Sigma \simeq 10^{(15-16)} \text{GeV}$ and $G_H G'_H \sim G'_H G_H \sim 1$, M_{H_c} and $M_{\bar{H}_c}$ are at $10^{(14-15)} \text{GeV}$. The four $SU(2)_L$ -doublet Higgses, H_f, \bar{H}_f, H'_f , and \bar{H}'_f , remain massless.

In order to break the PQ symmetry, we introduce a $SU(5)$ -singlet chiral multiplet P whose $U(1)_{PQ}$ charge is chosen as $P \rightarrow e^{-i(3\alpha+\beta)} P$ so that the following superpotential is allowed,

$$W'' = g_P \bar{H}'_A H'^A P. \quad (13)$$

The vacuum expectation value of P , which is constrained to be at M_{PQ} (see Eq. (1)), gives an intermediate-scale mass to a pair of $SU(2)_L$ -doublet Higgses, H'_f and \bar{H}'_f ,

$$M_{H'_f} = g_P \langle P \rangle. \quad (14)$$

The mechanism of breaking the PQ symmetry at the intermediate scale will be discussed in next section.

The color-triplet Higgses have an off-diagonal element in their mass matrix as

$$(\bar{H}_c, \bar{H}'_c) \begin{pmatrix} M_{\bar{H}_c} & 0 \\ g_P \langle P \rangle & M_{H_c} \end{pmatrix} \begin{pmatrix} H'_c \\ H_c \end{pmatrix}. \quad (15)$$

The baryon-number violating dimension-five operator mediated by the color-triplet Higgses is given in the present model as (see Fig. 1)

$$\frac{g_P \langle P \rangle}{M_{H_c} M_{\bar{H}_c}} \frac{1}{2\sqrt{2}} f_d^{ij} f_u^{kl} \left(\phi_{Fi} \psi_j^{(FA)} \right) \left(\psi_k^{(BC)} \psi_l^{(DE)} \right) \epsilon_{ABCDE}. \quad (16)$$

Notice that the pre-factor of the dimension-five operator in the original missing-

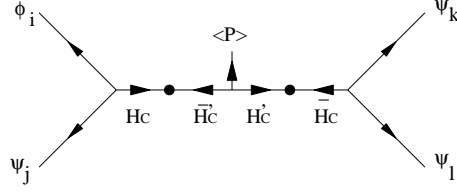


Figure 1: The Feynman diagram of the baryon-number violating dimension-five operator in the present model.

partner model is $f_d^{ij} f_u^{kl} / 2\sqrt{2}M_{H_c}$. Thus, we easily see that the dimension-five operator in the present model is more suppressed by a factor $M_{H'_f}/M_{H_c}$, and

$$\frac{M_{H_c} M_{\bar{H}_c}}{M_{H'_f}} \simeq 10^{(18-20)} \text{GeV}, \quad (17)$$

if $M_{H'_f} = 10^{10} \text{GeV}$ and $M_{H_c} \sim M_{\bar{H}_c} \sim 10^{(14-15)} \text{GeV}$. This value is consistent with the present negative observation of proton decay[6, 7, 8], even if $\tan\beta (\equiv \langle H_f \rangle / \langle \bar{H}_f \rangle) = (50 - 60)$ in which region the intriguing idea of the Yukawa coupling unification $f^t = f^b = f^\tau$ at the GUT scale [13] is consistent.

Now we will point out that this extension is consistent with the gauge coupling unification. The mass spectrum above the PQ symmetry breaking scale contains four $SU(2)_L$ -doublets Higgses, and the success of the gauge coupling unification seems to be lost at first sight. However, $\Sigma(\mathbf{75})$ gives large threshold corrections to the three gauge coupling constants as we mentioned before, and the corrections can compensate those from the extra $SU(2)_L$ -doublet Higgses. The running of the three gauge coupling constants at the one-loop level is given by the following solutions to the renormalization group equations,

$$\begin{aligned} \alpha_3^{-1}(m_Z) &= \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi} \left\{ \left(-2 - \frac{2}{3}N_g \right) \ln \frac{m_{SUSY}}{m_Z} + (-9 + 2N_g) \ln \frac{\Lambda}{m_Z} \right. \\ &\quad \left. -4 \ln \frac{\Lambda}{M_V} + \ln \frac{\Lambda}{M_{H_c}} + \ln \frac{\Lambda}{M_{\bar{H}_c}} \right. \\ &\quad \left. + 9 \ln \frac{\Lambda}{M_\Sigma} + \ln \frac{\Lambda}{0.8M_\Sigma} + 10 \ln \frac{\Lambda}{0.4M_\Sigma} + 3 \ln \frac{\Lambda}{0.2M_\Sigma} \right\}, \quad (18) \\ \alpha_2^{-1}(m_Z) &= \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi} \left\{ \left(-\frac{4}{3} - \frac{2}{3}N_g - \frac{5}{6} \right) \ln \frac{m_{SUSY}}{m_Z} + (-6 + 2N_g + 1) \ln \frac{\Lambda}{m_Z} \right\} \end{aligned}$$

$$\begin{aligned}
& -6 \ln \frac{\Lambda}{M_V} + \ln \frac{\Lambda}{M_{H'_f}} \\
& + 16 \ln \frac{\Lambda}{M_\Sigma} + 6 \ln \frac{\Lambda}{0.4 M_\Sigma} \Big\}, \tag{19}
\end{aligned}$$

$$\begin{aligned}
\alpha_1^{-1}(m_Z) = & \alpha_5^{-1}(\Lambda) + \frac{1}{2\pi} \Big\{ \left(-\frac{2}{3} N_g - \frac{1}{2} \right) \ln \frac{m_{SUSY}}{m_Z} + \left(2N_g + \frac{3}{5} \right) \ln \frac{\Lambda}{m_Z} \\
& - 10 \ln \frac{\Lambda}{M_V} + \frac{2}{5} \ln \frac{\Lambda}{M_{H_c}} + \frac{2}{5} \ln \frac{\Lambda}{M_{\overline{H}_c}} + \frac{3}{5} \ln \frac{\Lambda}{M_{H'_f}} \\
& + 10 \ln \frac{\Lambda}{0.8 M_\Sigma} + 10 \ln \frac{\Lambda}{0.4 M_\Sigma} \Big\}, \tag{20}
\end{aligned}$$

where $\alpha_5 \equiv g_5^2/4\pi$ is the SU(5) gauge coupling constant, M_V the heavy gauge boson mass ($M_V = 2\sqrt{15}g_5 V_\Sigma$), and Λ the renormalization point which is taken to be much larger than the GUT scale. Here, we have assumed that all superparticles in MSSM have a SUSY-breaking common mass m_{SUSY} for simplicity, and the mass splitting of $\Sigma(\mathbf{75})$ has been included. By eliminating α_5^{-1} from Eqs. (18-20), we obtain simple relations [7, 9, 11]:

$$\begin{aligned}
(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(m_Z) = & \frac{1}{2\pi} \Big\{ \frac{12}{5} \ln \frac{M_{H_c} M_{\overline{H}_c}}{M_{H'_f} m_Z} - 2 \ln \frac{m_{SUSY}}{m_Z} \\
& - \frac{12}{5} \ln(1.7 \times 10^4) \Big\}, \tag{21}
\end{aligned}$$

$$\begin{aligned}
(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_Z) = & \frac{1}{2\pi} \Big\{ 12 \ln \frac{M_V^2 M_\Sigma}{m_Z^3} + 8 \ln \frac{m_{SUSY}}{m_Z} \\
& + 36 \ln(1.4) \Big\}. \tag{22}
\end{aligned}$$

Notice that the last terms in Eqs. (21,22) come from the mass splitting of $\Sigma(\mathbf{75})$, which makes a crucial difference between the present model and the extension of the minimal SU(5) SUSY GUT where **24**-dimension Higgs breaks the SU(5) symmetry.

To perform a quantitative analysis, we use the two-loop renormalization group equations between the weak and the GUT scales. Instead of the common mass m_{SUSY} of superparticles we have used the mass spectrum estimated from the minimum supergravity [7, 14] to calculate the one-loop threshold correction at the SUSY-breaking scale. Using the experimental data $\alpha^{-1}(m_Z) = 127.9 \pm 0.1$, $\sin^2 \theta_W(m_Z) =$

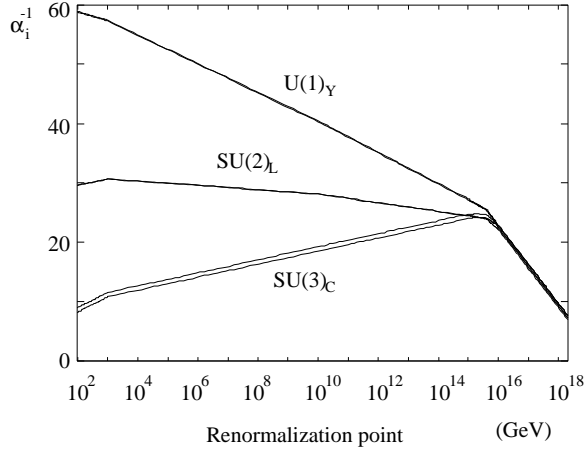


Figure 2: The flows of the running gauge coupling constants of $SU(3)_C \times SU(2)_L \times U(1)_Y$ and $SU(5)$. Here, M_{H_c} and $M_{\overline{H}_c}$ are taken at 10^{15}GeV , and $M_{H'_f}$ at 10^{10}GeV . We assume the SUSY-breaking scale $\sim 1\text{TeV}$.

0.2315 ± 0.0003 , and $\alpha_3(m_Z) = 0.116 \pm 0.005$ [15], we obtain

$$1.9 \times 10^{17} \text{ GeV} \leq \frac{M_{H_c} M_{\overline{H}_c}}{M_{H'_f}} \leq 1.3 \times 10^{20} \text{ GeV}, \quad (23)$$

$$9.1 \times 10^{15} \text{ GeV} \leq (M_V^2 M_\Sigma)^{1/3} \leq 1.7 \times 10^{16} \text{ GeV}. \quad (24)$$

The value of Eq. (23) is very much consistent with Eq. (17). Notice that this reason comes from the presence of the constant term in Eq. (21) which originates from the mass splitting of $\Sigma(\mathbf{75})$.

We show the evolution of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ and the $SU(5)$ gauge coupling constants in Fig. 2 taking $M_{H_c} = M_{\overline{H}_c} = 10^{15}\text{GeV}$ and $M_{H'_f} = 10^{10}\text{GeV}$ for a demonstrational purpose. The unification of the three gauge coupling constants occurs around 10^{16}GeV and the $SU(5)$ gauge coupling constant stays in the perturbative regime below the gravitational scale, $2.4 \times 10^{18}\text{GeV}$.

3 Conclusions and Discussion

The Peccei-Quinn symmetric extension of the missing-partner model in SUSY $SU(5)$ GUT is consistent with the observed stability of the proton, even if the masses of the unwanted **50**-dimension Higgses are lifted up to the gravitational scale so that the $SU(5)$ gauge coupling constant remains small enough for the perturbative

description of GUT's. Moreover, even the large $\tan\beta$ region, expected from the Yukawa unification, is allowed from the experimental constraint. Here, we will comment some points for this model.

First, so far we have assumed that P acquires the vacuum expectation value at M_{PQ} without the explicit potential. This is possible if we introduce another SU(5)-singlet chiral multiplet Q , whose U(1)_{PQ} charge is chosen as $Q \rightarrow e^{3i(3\alpha+\beta)}Q$ so that the following superpotential is allowed [16],

$$W''' = \frac{f}{M} P^3 Q. \quad (25)$$

These Higgses, P and Q , have a very flat scalar potential as

$$V(P, Q) = \frac{f^2}{M^2} |P|^6 + \frac{f^2}{M^2} |3P^2 Q|^2. \quad (26)$$

If a negative soft SUSY breaking mass $\sim -m^2$ for P is introduced by any strong Yukawa coupling, it can induce very naturally the PQ symmetry breaking [16],

$$\langle P \rangle \simeq \langle Q \rangle \simeq \sqrt{\frac{Mm}{f}} \sim 10^{11} \text{GeV}, \quad (27)$$

provided $m \sim 1 \text{TeV}$ and $f \sim 1$.

Next, two independent charges α and β defined in Eqs. (4,6) mean the presence of two global U(1) symmetries. If we introduce right-handed neutrino multiplets N_i (1) and add the following terms to the superpotential,

$$W'''' = k_{ij} N_i \phi_j H + j_{ij} N_i N_j P, \quad (28)$$

we have only one U(1) symmetry and the charge α is fixed as $\alpha = 3\beta$ [17]. The Yukawa couplings $j_{ij} N_i N_j P$ in Eq. (28) induce Majorana masses for the right-handed neutrino multiplets N_i , with $\langle P \rangle \neq 0$. It is interesting that the Majorana masses for the right-handed neutrinos are expected to be $O(10^{11})$ GeV, which naturally induce very small masses of neutrinos through the celebrated see-saw mechanism [18] in a range of the MSW solution [19] to the solar neutrino problem.

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